Reply to ‘Comment on “The influence of cosmological transitions on the evolution of density perturbations” ’

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We reply to L.P. Grishchuk’s comment [1] on our paper “The Influence of Cosmological Transitions on the Evolution of Density Perturbations” [2]. We show that his points of criticism are not correct.

Inflationary cosmology predicts the generation of density perturbations and gravitational waves with almost scale-invariant spectra. The ratio between their amplitudes (at superhorizon scales) is fixed by the equation of state during inflation. The closer the inflationary epoch is to a de Sitter phase, the larger are the generated density perturbations with respect to the gravitational waves. This statement is the so-called “standard result”. The reason for this result is that the gauge-invariant metric perturbation Φ grows by a huge factor during reheating, whereas the gravitational waves amplitude $h_{\rm gw}$ essentially remains constant during that epoch.

Four years ago Grishchuk published a paper [3] in which he stated that there is no big amplification of density perturbations. As a consequence, he expressed his disagreement with the standard result. This claim was criticized by Deruelle and Mukhanov [4]. They wrote that Grishchuk had not properly taken the joining conditions at reheating into account. A central point of our paper [2] is that Grishchuk’s criticism was wrong indeed (and that the standard result is correct), not because of the joining conditions, though. The origin of Grishchuk’s error was that he had not evolved the perturbations through reheating correctly.

Let us draw our attention on four issues that are discussed in Grishchuk’s comment:

1. Sharp transition: Grishchuk defines a new quantity $\bar{\mu} := \mu/\sqrt{\gamma}$. The introduction of this new variable $\bar{\mu}$ does not add anything new to the problem. In particular, it does not alter the behavior of the growing superhorizon mode $\mu \propto a\sqrt{\gamma}$, see the equation before Eq. (14) in Ref. [1].

Below Eq. (17) of Ref. [1] Grishchuk states that $\bar{\mu}$ is continuous through a sharp transition of the equation of state. This is true and we have used this condition in Eq. (6.5) of our paper. In addition, Grishchuk writes that “It was shown [3,5] that Eq. (9) requires the continuity of the function v, where $v = \gamma(\bar{\mu}/a)'$ and, hence, the continuity of the function $\bar{\mu}/a$.” and “This evolution is at the basis of practical calculations in [3,5,6]”. This is in fact in total contradiction to the “practical calculations” done in the quoted Ref. [3]. Firstly, the quantity $\bar{\mu}$ is not even defined in that paper and therefore cannot have been used. Secondly, one of the equations after Eq. (48) (on p. 7161) is:

$$\mu|_{\eta=\eta_1-0} = \mu|_{\eta=\eta_1+0}. \tag{1}$$

Therefore, it is clear that Grishchuk used the continuity of μ and not that of $\bar{\mu}$. Now Grishchuk says that this equation is a misprint [7], which we do not believe. On the contrary, we think that this is the origin of the error as it is explained in our paper. If one uses the continuity of μ (mistakenly), then one loses the factor $\sqrt{\gamma}$, which is responsible for the large amplification. This was exactly the mistake in Ref. [3].

2. Constancy of ζ: The arguments in [1] concerning the constancy of ζ are inconsistent. Grishchuk writes that “The mysterious ‘ζ conservation law’ has only that meaning that the ‘growing’ part of the function $\bar{\mu}/a$, in its lowest nonvanishing long-wavelength approximation, is a constant ...”. This is confirmed by Eq. (4.21) of our paper and is one of the conclusions of our article. If the decaying mode is neglected, one concludes that $\zeta = -\bar{A}_1/2 = cte.$ ζ is a constant and is determined from the initial conditions. But ten lines later, Grishchuk writes the contrary: “It was shown [5,6] that if you dare to respect the original Einstein equations, the constant (23), must be a strict zero, and not a constant determined by initial conditions”!

Obviously, one is free to use other methods than the ζ argument to calculate the evolution of the perturbations. But if one chooses to use the constancy of ζ, the result will be correct (see Ref. [2]).

3. Big amplification: Grishchuk’s words do not say the same as his equations. He says that “big amplification is a misinterpretation” . But, ironically, he himself derives the standard result (as it is obvious if one calculates the ratio of the Eqs. (33) and (36) of Ref. [1])! We

do not understand how Grishchuk can say, at the same time, that the standard result is wrong and that he agrees with Eq. (4.25) of our work [2] (bottom of p. 10), since this equation is precisely the standard result!

Grishchuk gives an example to show that if the amplification coefficient is big, it does not follow that the real (absolute) value of $\Phi(\eta_m)$ is big also. Again this remark is out of place. We wrote exactly the same after Eq. (4.7) in our article. In order to determine the absolute value one needs to specify the initial conditions.

4. Quantum initial conditions: Finally, the quantum normalization of density perturbations used on p. 12 of Ref. [1] is wrong. Apparently, in his comment Grishchuk changes his opinion and now accepts the discontinuity in μ but, in order to avoid the factor $\sqrt{\gamma}$ again, uses wrong initial conditions. Grishchuk claims that the scalar metric perturbation h in the synchronous gauge is $h(k) \approx (l_{\rm Pl}/l_0)k^{2+\beta}$. The last equation implies the asymptotic behavior $\lim_{k\to+\infty}\bar{\mu}(\eta,\mathbf{k}) = -4\sqrt{\pi}l_{\rm Pl}e^{-ik(\eta-\eta_0)}/\sqrt{2k}$. Together with the (correct) normalization of the scalar field $\lim_{k\to+\infty}\varphi_1(\eta,\mathbf{k}) = \sqrt{\hbar}e^{-ik(\eta-\eta_0)}/(a\sqrt{2k})$ (see Sec. VII of Ref. [3]), the normalization of $\bar{\mu}$ is in contradiction to the Einstein equations. The normalization of the scalar field fixes the normalization of density perturbations via the $0-i$ component of the Einstein equations:

$$\lim_{k\to+\infty}\mu(\eta,\mathbf{k}) = -\sqrt{2\kappa}a\varphi_1(\eta,\mathbf{k}) \ . \qquad (2)$$

Since μ appears in this equation, and not $\bar{\mu}$, this leads to the following correct asymptotic behavior (as it is demonstrated in the appendix of our paper [2]): $\lim_{k\to+\infty}\mu(\eta,\mathbf{k}) = -4\sqrt{\pi}l_{Pl}e^{-ik(\eta-\eta_0)}/\sqrt{2k}$, which implies $h(k) \approx (l_{\rm Pl}/l_0)k^{2+\beta}/\sqrt{\gamma}$. Again, Grishchuk is wrong by a factor $\sqrt{\gamma}$.

In conclusion, Grishchuk's points of criticism [1] on our paper [2] are incorrect. His remarks are either inconsistent or an (involuntary) confirmation of the results obtained in Ref. [2]. We consider this controversy to be settled now.

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[6] L. P. Grishchuk, in *String Gravity and Physics at the Planck Energy Scale*, Eds. N. Sanchez and A. Zichichi (Kluwer, Dordrecht, 1996) p. 369.

[7] Private communication.